Diffractive neutrino-production of pions in the color dipole model

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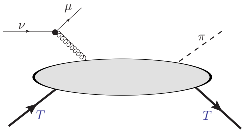
Deep Inelastic Scattering 2011
Newport News, Virginia
April 12, 2011

Outline

1. Overview
   - Process & kinematics
   - Adler relation and beyond
2. Evaluation in color dipole model
   - Evaluation on the proton target
   - Evaluation on the nuclear target

Kinemaitcs

- Diffractive pion production, $\nu T \to \mu \pi T$
  - T is either proton or nucleus
  - neutrino may be ν_μ, ν_e

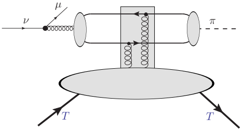


$$E_\nu = \frac{p\cdot k_\nu}{m_N},\ \nu = \frac{p\cdot q_W}{M},\ y = \frac{p\cdot q_W}{p\cdot k}$$

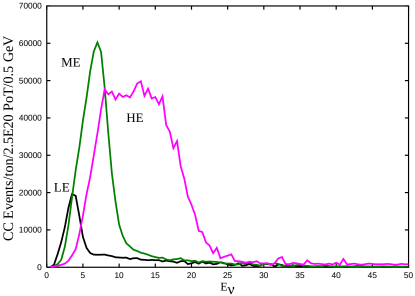
$$Q^2 = -q_W^2 = 4E_\nu(E_\nu - \nu)\sin^2\frac{\theta}{2} + \mathcal{O}\left(m_l^2\right)$$

$$t = (p' - p)^2 = \Delta^2 = t_{min} - \Delta_\perp^2$$

Kinemaitcs



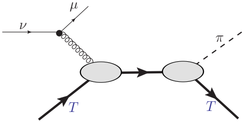
- Diffractive pion production, $\nu T \to \mu \pi T$
- Diffractive kinematics, energy $\nu \gg \nu_{min} \sim (Q^2 + m_\pi^2) R_A$
  Minerva@Fermilab:



- High statistics ($\sim 2.5 \times 10^{20}$ PoT/year)
- Differential cross-sections are measured
- Proton and nuclear targets (C, Fe, Pb)

Kinemaitcs

- Diffractive pion production, $\nu T \to \mu \pi T$
- Diffractive kinematics, energy $\nu \gg \nu_{min} \sim (Q^2 + m_\pi^2) R_A$
- In the small-ν dominant contribution comes from resonances



Adler relation & PCAC

- PCAC Hypothesis: Operator relation,

$$\partial_\mu A_\mu \sim m_\pi^2 \phi_\pi(x).$$

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- For the case of small $q^2 \approx m_l^2 \approx 0$ and $k_\mu \sim q_\mu$, so lepton tensor may be cast to the form

$$L_{\mu\nu} = 2\frac{E_\nu \left(E_\nu - \nu\right)}{\nu^2} q_\mu q_\nu + \mathscr{O}\left(q^2\right) + \mathscr{O}\left(m_l^2\right)$$

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- So the cross-section may be evaluated using the PCAC hypothesis (*S. Adler, 1966*)

$$\left.\frac{d\sigma_{\nu T \to lF}}{d\nu dQ^2}\right|_{Q^2=0} = \frac{G_F^2}{2\pi} f_\pi^2 \frac{E_\nu - \nu}{E_\nu \nu} \sigma_{\pi T \to F}$$

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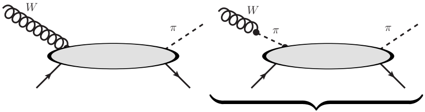
- In addition, we have contributions from transverse part and from the vector part ($\mathcal{O}\left(q^2\right)$ for small q^2)

Black disk limit

Adler relation is inconsistent with black disk limit: consider single-pion production,

$$\underbrace{\left.\frac{d\sigma_{\nu T\to l\pi T}}{d\nu\, dQ^2}\right|_{Q^2=0}}_{\text{off-forward diffraction, } W\to\pi} = \frac{G_F^2}{2\pi} f_\pi^2 \frac{E_\nu-\nu}{E_\nu \nu} \underbrace{\sigma_{\pi T\to\pi T}}_{\text{elastic scattering, } \pi\to\pi}$$



$$\sim \frac{q_\mu}{q^2-m_\pi}$$

(pions do not contribute due to lepton current conservation)

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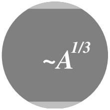
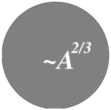




$\sim 2\pi R$ $\qquad$ $\sim \pi R^2$

Energy dependence in limit $s\to\infty$:

$\sim \ln s$ $\qquad$ $\sim \ln^2 s$

PCAC vs. pion dominance

Adler relation: replace W with π for $Q^2 = 0$

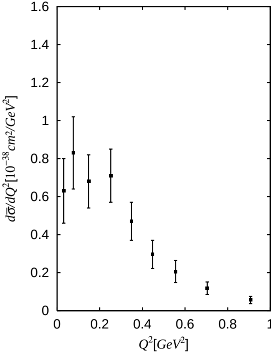
$$\left.\frac{d\sigma_{\nu T\to lF}}{d\nu dQ^2}\right|_{Q^2=0} = \frac{G_F^2}{2\pi} f_\pi^2 \frac{E_\nu - \nu}{E_\nu \nu} \sigma_{\pi T\to F}$$

Pion dominance model:

$$T_\mu(\ldots) \sim \frac{q_\mu}{q^2 - m_\pi^2} + T_\mu^{non-pion}(\ldots),$$

but lepton currents are conserved, so

$$q_\mu L_{\mu\nu} = \mathcal{O}(m_l)$$



$\Rightarrow$contribution of pions is zero

Barish et. al, 1979

$\Rightarrow$contribution of non-pions should exactly match the contribution of pions

Chiral symmetry & PCAC

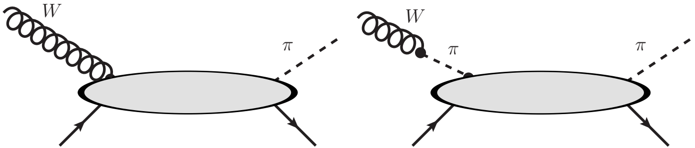


Figure: W may couple directly to quarks in the target or via intermediate pion

$$\mathscr{L}_2 \approx \frac{F^2}{2}\left(\partial_\mu\vec{\phi} - \vec{a}_\mu\right)^2 + \mathscr{O}\left(m, \phi^3, a^3, a^2\phi, \dots\right),$$

$$\mathscr{L}^{(1)}_{\pi N} \approx \bar{\Psi}\left(i\gamma_\mu\partial_\mu + m_N - i\frac{g_A}{4}\gamma_\mu\gamma_5\left(\vec{a}_\mu - \partial_\mu\vec{\phi}\right)\right)\Psi + \mathscr{O}\left(m, \phi^3, a^3, a^2\phi, \dots\right).$$

$$T_\mu^{(a\to\pi)} = T_{\pi\pi}(p,q)\left(\frac{q_\mu q_\nu}{q^2 - m_\pi^2} - g_{\mu\nu}\right)P_\nu\left(p, \Delta\right),$$

Chiral symmetry & PCAC & color dipole

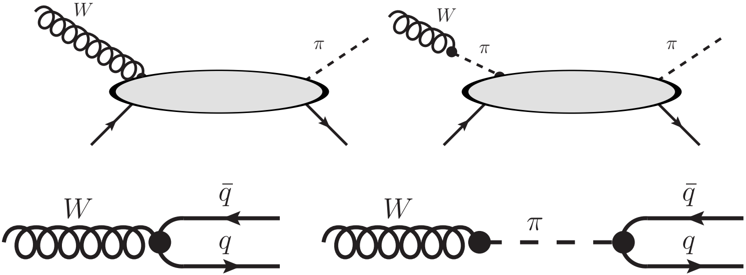
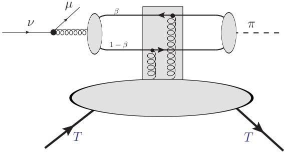


Figure: Relation between couplings $\pi\bar{q}q$, $W\bar{q}q, W\pi$ guraantees that the amplitude remains transverse

$$T_{\mu}^{(a\to\pi)}=T_{\pi\pi}(p,q)\left(\frac{q_{\mu}q_{\nu}}{q^{2}-m_{\pi}^{2}}-g_{\mu\nu}\right)P_{\nu}\left(p,\Delta\right),$$

Color dipole and neutrino-proton interactions



The amplitude has a form

$$\mathscr{A}^{aT\to\pi T}=\int d\beta\, d\beta' d^2r d^2r' \bar{\Psi}_\pi\left(\beta',r'\right)\mathscr{A}_T^d\left(\beta',r';\beta,r\right)\Psi_a\left(\beta,r\right),$$

- $\mathscr{A}_T^d\left(\beta',r';\beta,r\right)$ universal object, depends only on the target T. Known from photon-proton and photon-nuclear processes
  - Universal (depends on the target)
  - In the small-r limit behaves like $\mathscr{A}^d\left(\beta,r\right)\sim r^2$
  - Its evolution is described by BK equation (*Balitsky 1995; Kovchegov 1999*)
  - There are soft contribiutions, which correspond to large-size dipoles.
- $\bar{\Psi}_\pi,\Psi_a$ are the distribution amplitudes of the initial and final states

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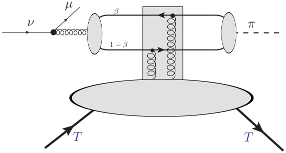

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- $\bar{\Psi}_\pi,\Psi_a$ are the distribution amplitudes of the initial and final states
- Earlier applications of color dipole model:
  - Formulated for photon-proton and proton-nuclear processes (vector current)
  - Applications to processes with neutrinos (vector current)
    - ★ electroweak DVCS (*Machado 2007*)
    - ★ electroweak DIS (*Fiore, Zoller 2005; Gay Ducati, Machado 2007*)
    - ★ charm/heavy meson production (*Fiore, Zoller 2009; Gay Ducati, Machado 2009*)

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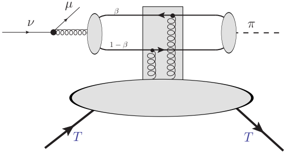

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- We are going to use color dipole for description of the axial current

Extension from vector to axial current

Extension of effective models from vector to axial current is not straightforward.
Example: extension of Generalized Vector meson Dominance (GVMD) leads to Piketty-Stodolsky paradox:

$$\sigma_{\pi p\to\pi p} \neq \sigma_{\pi p\to a_1 p}$$

- VMD does not work for axial current, dominant contributions comes from multimeson states $(\rho\pi, \pi\pi\pi, \ldots)$ (*Belkov, Kopeliovich, 1986*)

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- In color dipole we do not have such problems since there is no explicit hadrons like in GVMD

Distribution amplitudes from the Instanton Vacuum Model

Why IVM ?

- The model is valid for low virtualities Q^2

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- The model is valid for low virtualities Q^2
- The model has built-in chiral symmetry
- Effective action :

$$S = \int d^4x \left(2\Phi^\dagger(x)\Phi(x) - \bar{\psi}\left(\hat{p} + \hat{v} + \hat{a}\gamma_5 - m - c\bar{L}f \otimes \Phi \cdot \Gamma_m \otimes fL\right)\psi\right),$$

  - has only two parameters (average instanton size $\rho \sim 1/600 MeV$ and average distance $R \sim 1/200 MeV$), but reproduces the low-energy constants in chiral lagrangian.
  - may be rewritten as NJL with nonlocal interactions (nonlocality from instanton shape)

Distribution amplitudes of pion

Pion distribution amplitudes (P. Ball *et al*, 2006)

$$\begin{aligned}\left\langle 0\left|\bar{\psi}(y)\gamma_\mu\gamma_5\psi(x)\right|\pi(q)\right\rangle &= if_\pi\int_0^1 du\, e^{i(up\cdot y+\bar{u}p\cdot x)}\times\\ &\times\left(p_\mu\phi_{2;\pi}(u)+\frac{1}{2}\frac{z_\mu}{(p\cdot z)}\psi_{4;\pi}(u)\right),\end{aligned}$$

$$\langle 0|\bar{\psi}(y)\gamma_5\psi(x)|\pi(q)\rangle = -if_\pi\frac{m_\pi^2}{m_u+m_d}\int_0^1 du\, e^{i(up\cdot y+\bar{u}p\cdot x)}\phi_{3;\pi}^{(p)}(u).$$

$$\begin{aligned}\left\langle 0\left|\bar{\psi}(y)\sigma_{\mu\nu}\gamma_5\psi(x)\right|\pi(q)\right\rangle &= -\frac{i}{3}f_\pi\frac{m_\pi^2}{m_u+m_d}\int_0^1 du\, e^{i(up\cdot y+\bar{u}p\cdot x)}\times\\ &\times\frac{1}{p\cdot z}\left(p_\mu z_\nu-p_\nu z_\mu\right)\phi_{3;\pi}^{(\sigma)}(u),\end{aligned}$$

Distribution amplitudes of axial meson

Axial distribution amplitudes (K.-C. Yang 2007)

$$\begin{aligned}\left\langle 0\left|\bar\psi(y)\gamma_\mu\gamma_5\psi(x)\right|A(q)\right\rangle &= if_A m_A\int_0^1 du\, e^{i(up\cdot y+\bar u p\cdot x)}\times\\ &\times\left(p_\mu\frac{e^{(\lambda)}\cdot z}{p\cdot z}\Phi_{||}(u)+e_\mu^{(\lambda=\perp)}g_\perp^{(a)}(u)-\frac{1}{2}z_\mu\frac{e^{(\lambda)}\cdot z}{(p\cdot z)^2}m_A^2 g_3(u)\right),\end{aligned}$$

$$\left\langle 0\left|\bar\psi(y)\gamma_\mu\psi(x)\right|A(q)\right\rangle = -if_A m_A\varepsilon_{\mu\nu\rho\sigma}e_\nu^{(\lambda)}p_\rho z_\sigma\int_0^1 du\, e^{i(up\cdot y+\bar u p\cdot x)}\frac{g_\perp^{(v)}(u)}{4}$$

$$\begin{aligned}\left\langle 0\left|\bar\psi(y)\sigma_{\mu\nu}\gamma_5\psi(x)\right|A(q)\right\rangle &= f_A^\perp\int_0^1 du\, e^{i(up\cdot y+\bar u p\cdot x)}\left(e_{[\mu}^{(\lambda=\perp)}p_{\nu]}\Phi_\perp(u)\right.\\ &+\left.\frac{e^{(\lambda)}\cdot z}{(p\cdot z)^2}m_A^2 p_{[\mu}z_{\nu]}h_{||}^{(t)}(u)+\frac{1}{2}e_{[\mu}^{(\lambda)}z_{\nu]}\frac{m_A^2}{p\cdot z}h_3(u)\right),\end{aligned}$$

$$\langle 0|\bar\psi(y)\gamma_5\psi(x)|A(q)\rangle = f_A^\perp m_A^2 e^{(\lambda)}\cdot z\int_0^1 du\, e^{i(up\cdot y+\bar u p\cdot x)}\frac{h_{||}^{(p)}(u)}{2}.$$

Result for the $\nu p \to \mu^- \pi^+ p$ cross-section

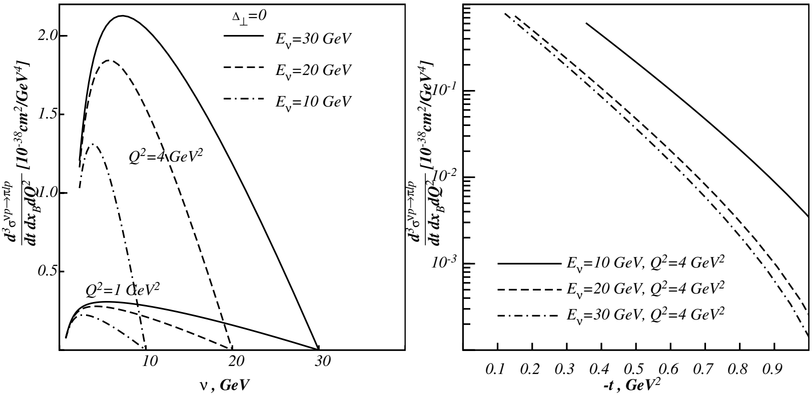


Figure: Differential cross-section $d\sigma/d\nu dt dQ^2$ for different neutrino energies E_ν.

Result for the $\nu p \to \mu^- \pi^+ p$ cross-section

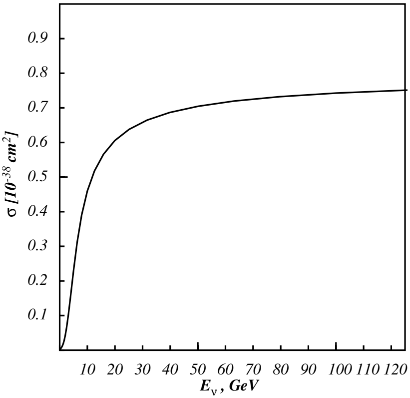


Figure: Total cross-section as a function of the neutrino energy E_ν.

Result for the $\nu p \to \mu^- \pi^+ p$ cross-section

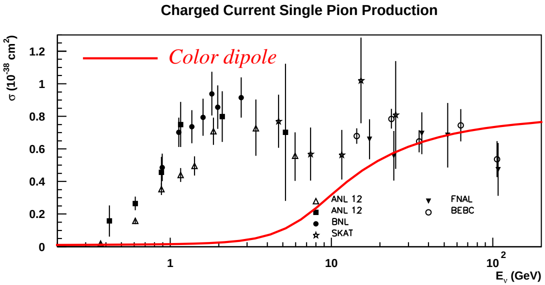
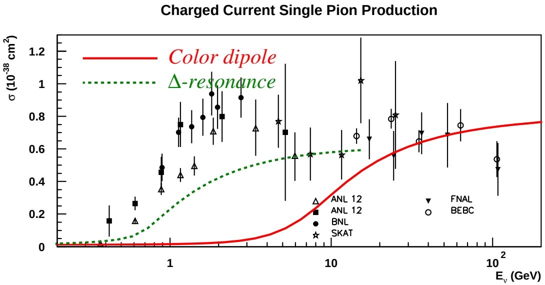


Figure: Total cross-section as a function of the neutrino energy E_ν.*Compilation of experimental data from Minerva proposal, 2004*

Agreement for energies $E_\nu > 10$ GeV, problem for $E_\nu < 10$ GeV

Result for the $\nu p \to \mu^- \pi^+ p$ cross-section



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Low-energy region is dominated by Δ

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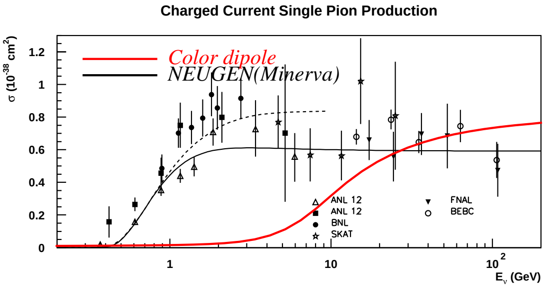

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Difference between NEUGEN and color dipole: cross-section is slowly growing for high energies

Coherent neutrino-nuclear scattering

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and coherence length of the effective axial meson state,

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- For small $m_\pi^2 \lesssim Q^2 \ll m_a^2$ the two scales are essentially different, $l_c^a \ll l_c^\pi$, so there are three regimes depending on relations between R_A and l_c^a, l_c^π.

Coherent neutrino-nuclear scattering (contd.)

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- $R_A \ll l_c^a \ll l_c^\pi$: absorptive corrections are large, Adler relation is not valid even for $Q^2 = 0$, $\sigma \sim A^{1/3}$

Result for the $\nu A \to l\pi^+ A$ differential cross-section

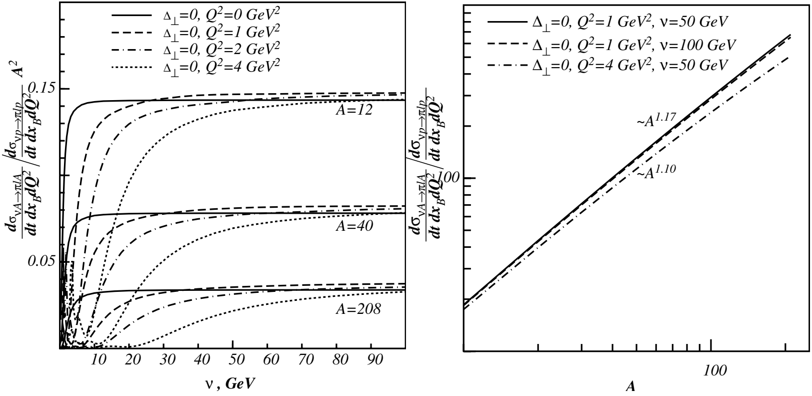

Figure: Ratio of cross-sections on the nucleus and proton.

$d\sigma_{\nu A\to\mu\pi A}/dtdxdQ^2 \sim A \Rightarrow \sigma_{\nu A\to\mu\pi A} \sim A^{1/3}$, but Adler relation requires $\sigma_{\nu A\to\mu\pi A} \sim \sigma_{\pi A} \sim A^{2/3} \Rightarrow$ for high energies the Adler relation is broken

Conclusion

- We have shown that the Adler relation cannot always be correct for the neutrino-nuclear processes–broken in black disk limit, by absorptive corrections.

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- We have shown that the Adler relation cannot always be correct for the neutrino-nuclear processes–broken in black disk limit, by absorptive corrections.
- We evaluated the results in color dipole model; for small-Q^2 and moderate energies we reproduce Adler relation; our results are valid also for $Q^2 \neq 0$ (and $\nu \gg m_N$)

- Thank You for your attention !

Absorptive corrections

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- For diffractive meson production they have a form

$$\sigma_{\pi A \to MA} \sim \int d^2 b \frac{\exp\left(-\frac{1}{2}\sigma_{el}^{\pi N} T_A(b)\right) - \exp\left(-\frac{1}{2}\sigma_{el}^{MN} T_A(b)\right)}{\sigma_{el}^{\pi N} - \sigma_{el}^{MN}} \approx$$
$$\approx \int d^2 b \exp\left(-\frac{1}{2}\sigma_{el}^{\pi N} T_A(b)\right)$$

-different in black disk limit

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$\Rightarrow$contribution of pions is zero$\Rightarrow$contribution of non-pions should exactly match to the contribution of pions

Chiral symmetry & PCAC

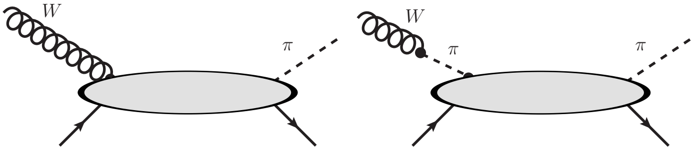

Figure: W may couple directly to quarks in the target or via intermediate pion

$$\mathcal{L}_2 \approx \frac{F^2}{2}\left(\partial_\mu\vec{\phi} - \vec{a}_\mu\right)^2 + \mathcal{O}\left(m, \phi^3, a^3, a^2\phi, \ldots\right),$$

$$\mathcal{L}_{\pi N}^{(1)} \approx \bar{\Psi}\left(i\gamma_\mu\partial_\mu + m_N - i\frac{g_A}{4}\gamma_\mu\gamma_5\left(\vec{a}_\mu - \partial_\mu\vec{\phi}\right)\right)\Psi + \mathcal{O}\left(m, \phi^3, a^3, a^2\phi, \ldots\right).$$

$$T_\mu^{(a\to\pi)} = T_{\pi\pi}(p,q)\left(\frac{q_\mu q_\nu}{q^2 - m_\pi^2} - g_{\mu\nu}\right)P_\nu\left(p,\Delta\right),$$

Chiral symmetry & PCAC & color dipole

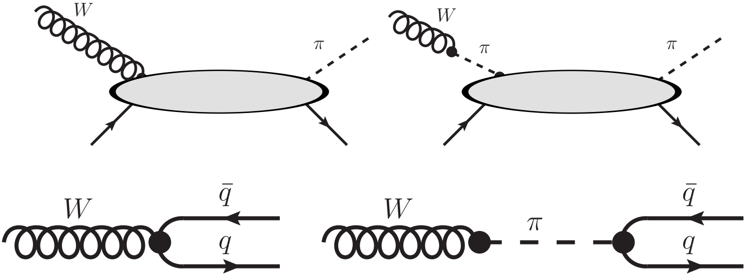

Figure: Relation between couplings $\pi\bar{q}q$, $W\bar{q}q, W\pi$ guraantees that the amplitude remains transverse

$$T_{\mu}^{(a\to\pi)} = T_{\pi\pi}(p,q)\left(\frac{q_{\mu}q_{\nu}}{q^2-m_{\pi}^2} - g_{\mu\nu}\right)P_{\nu}\left(p,\Delta\right),$$